Nonlinear Energy Harvesting

Brent Cook, Yuhao Pan, Joshua Paul, Luis Sanchez, Larissa Szwez, Joseph Tang

Our Problem

- Vibrational energy lost to environment
- Capturing wasted energy using inverted oscillator
- Analyze potential energy function to identify optimum physical parameters
- Helps to predict voltage generation

Potential Applications- Sensors

Bus Station Ticket Gates

thecityfix.com

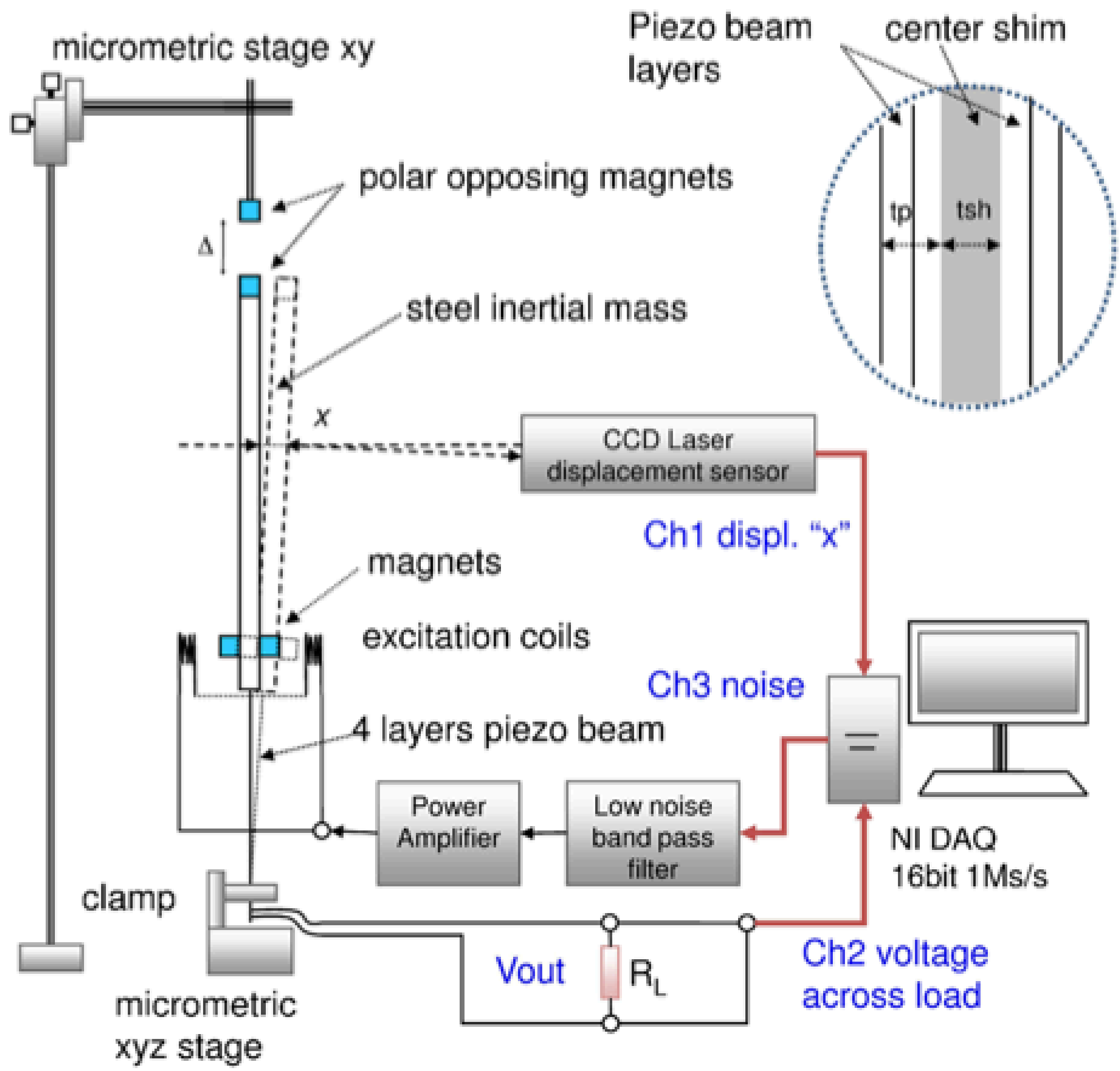
Sensors in Bridges

science.howstuffworks.com

Goals

- Maximize voltage harvested by the inverted oscillator
- Derive a model for the voltage produced by the system as a function of physical parameters
- Find the ideal combination of physical parameters to maximize the energy harvested

Physical Model



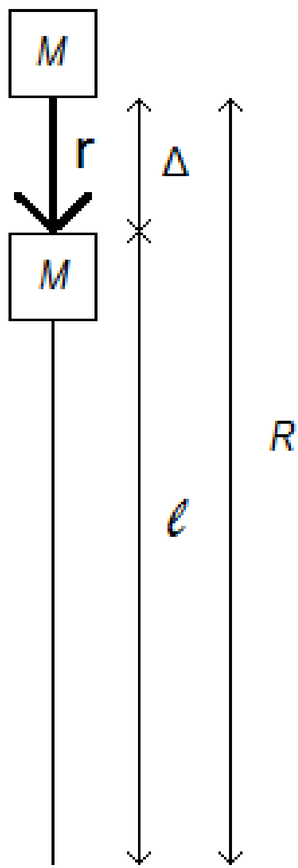
Theory: Equation of Motion and Voltage

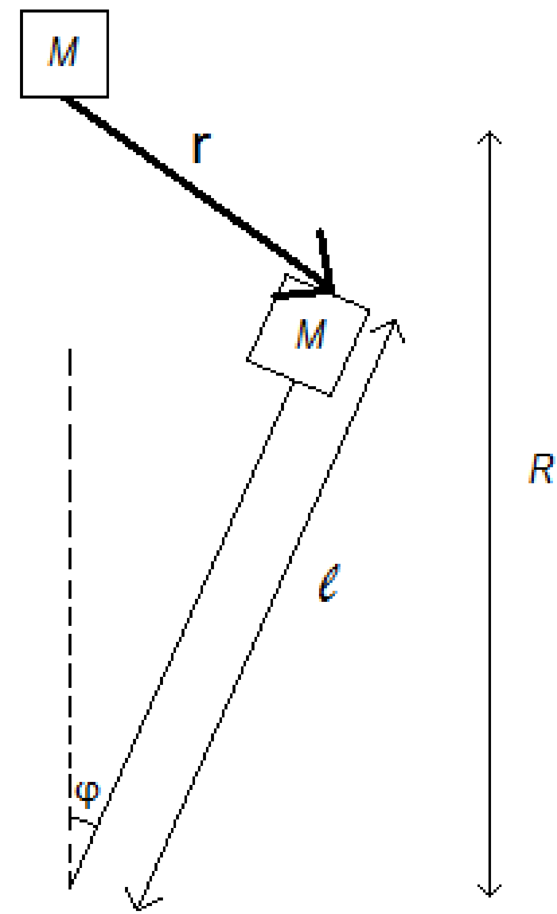
$$m_{eff}\ddot{\varphi} = \frac{dU(\varphi)}{d\varphi} - \gamma\dot{\varphi} - K_v V(t) + \sigma\varepsilon(t)$$

$$\dot{V}(t) = K_c\dot{\varphi} - \frac{V(t)}{R_L C}$$

Equation	Meaning
$m_{eff}\,\ddot{\varphi}$	The kinetic force of the oscillator
$U(\varphi)$	The potential energy function of the oscillator
$\gamma\dot{\varphi}$	Energy dissipation due to bending of the piezoelectric
K_vV(t)	Energy transferred to the resistor
$\sigma\varepsilon(t)$	Vibration force that drives the oscillations with stochastic process

Theory: Deriving Potential Energy





Theory: Deriving Potential Energy (Double Well)

$$\vec{l} = l * \sin(\varphi)\,\hat{x} + l * \cos(\varphi) * \hat{y}$$

$$\vec{F} = Q\,\frac{l * \sin(\varphi) * \hat{x} + (l * \cos(\varphi) - R) * \hat{y}}{(l^2 \sin^2(\varphi) + (l * \cos(\varphi) - R)^2)^{3/2}}$$

$$\overrightarrow{l_\perp} = l * \cos(\varphi)\,\hat{x} - l * \sin(\varphi) * \hat{y}$$

$$\overrightarrow{F_\perp} = \vec{F} * \frac{\overrightarrow{l_\perp}}{|\overrightarrow{l_\perp}|} = \frac{Q * R * \sin(\varphi)}{(l^2 + R^2 - 2lRcos(\varphi))^{3/2}}$$

$$\overrightarrow{F_{total}} = \overrightarrow{F_\perp} - K * \varphi$$

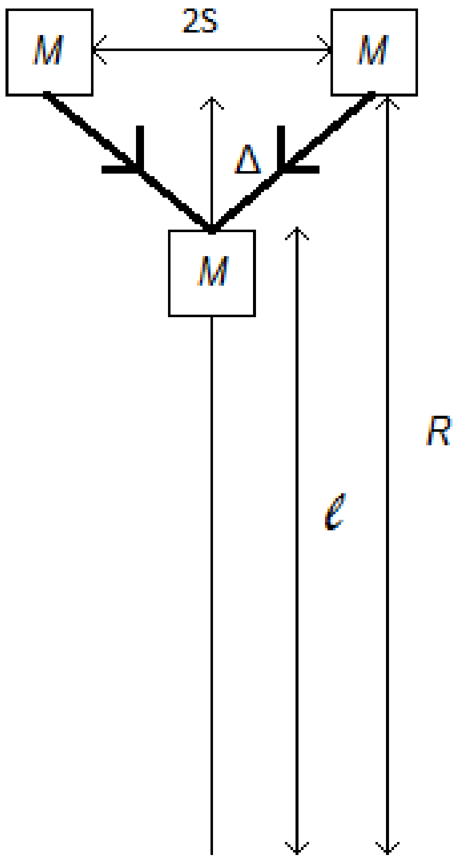
$$\overrightarrow{F_{total}} = \frac{-dU_{total}}{d\varphi}$$

Theory: Deriving Potential Energy (Double Well)

$$F(\varphi) = \frac{RQsin(\varphi)}{(L^2\sin^2(\varphi) + \left(R - Lcos(\varphi)\right)^2)^{3/2}} - k\varphi$$

$$U(\varphi) = k\frac{\varphi^2}{2} + \frac{Q}{L\sqrt{L^2 + R^2 - 2LRcos(\varphi)}}$$

Theory: Deriving Potential Function (Triple Well)



$$\vec{F} = \frac{Q}{2}\frac{(l*\sin(\varphi)+S)*\hat{x}+(l*\cos(\varphi)-R)*\hat{y}}{((l*sin(\varphi)+S)^2+(l*\cos(\varphi)-R)^2)^{3/2}} + \frac{Q}{2}\frac{(l*\sin(\varphi)-S)*\hat{x}+(l*\cos(\varphi)-R)*\hat{y}}{((l*sin(\varphi)-S)^2+(l*\cos(\varphi)-R)^2)^{3/2}}$$

$$\overrightarrow{F_\perp} = \vec{F} * \overrightarrow{l_\perp}/\left|\overrightarrow{l_\perp}\right| = \frac{Q/_2*(R*\sin(\varphi)\ +S*\cos(\varphi))}{(l^2+R^2+S^2-2l(R*cos(\varphi)+S*sin(\varphi)))^{3/2}} + \frac{Q/_2*(R*\sin(\varphi)-S*\cos(\varphi))}{(l^2+R^2+S^2-2l(R*cos(\varphi)-S*sin(\varphi)))^{3/2}}$$

$$\overrightarrow{F_{total}} = \overrightarrow{F_\perp} - K*\varphi$$

$$\overrightarrow{F_{total}} = \frac{-dU_{total}}{d\varphi}$$

Theory: Deriving Potential Function (Triple Well)

$$F(\varphi) = -K\varphi + \frac{{}^{Q}\!/_{2} * (R * \sin(\varphi) + S * \cos(\varphi))}{(l^2 + R^2 + S^2 - 2l(R * cos(\varphi) + S * sin(\varphi)))^{3/2}} + \frac{{}^{Q}\!/_{2} * (R * \sin(\varphi) - S * \cos(\varphi))}{(l^2 + R^2 + S^2 - 2l(R * cos(\varphi) - S * sin(\varphi)))^{3/2}}$$

$$U(\varphi) = \frac{K}{2}\varphi^2 + \frac{{}^{Q}\!/_{2}}{1 * \sqrt{l^2 + R^2 + S^2 + 2l(-R * \cos(\varphi) + S * \sin(\varphi))}} + \frac{{}^{Q}\!/_{2}}{1 * \sqrt{l^2 + R^2 + S^2 + 2l(-R * \cos(\varphi) - S * \sin(\varphi))}}$$

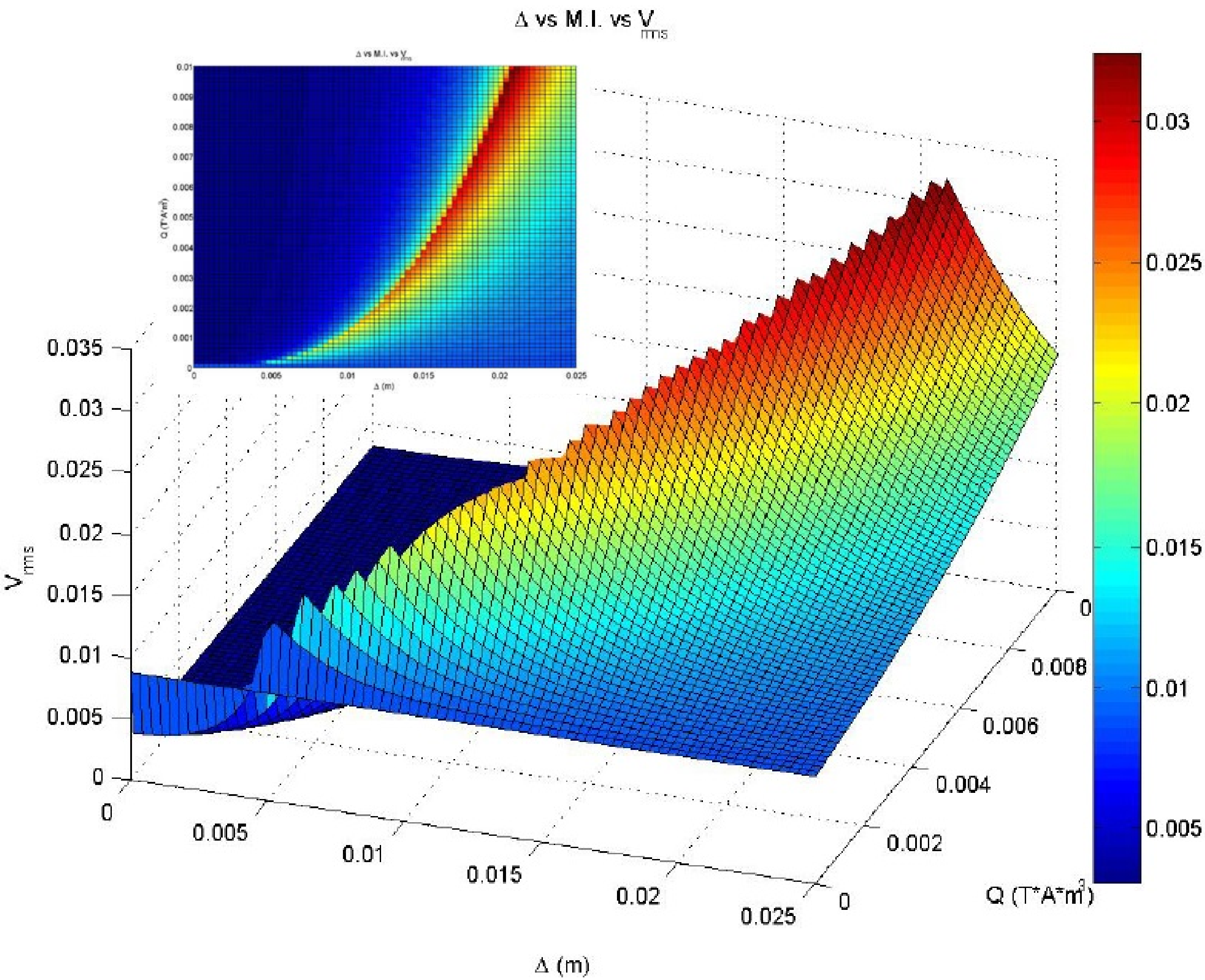
Calculating Voltage

- Voltage varies like an AC current
- $V_{rms} = \sqrt{\frac{\left(\sum_1^n V(n)^2\right)}{n^2}}$
- Self-Averaging

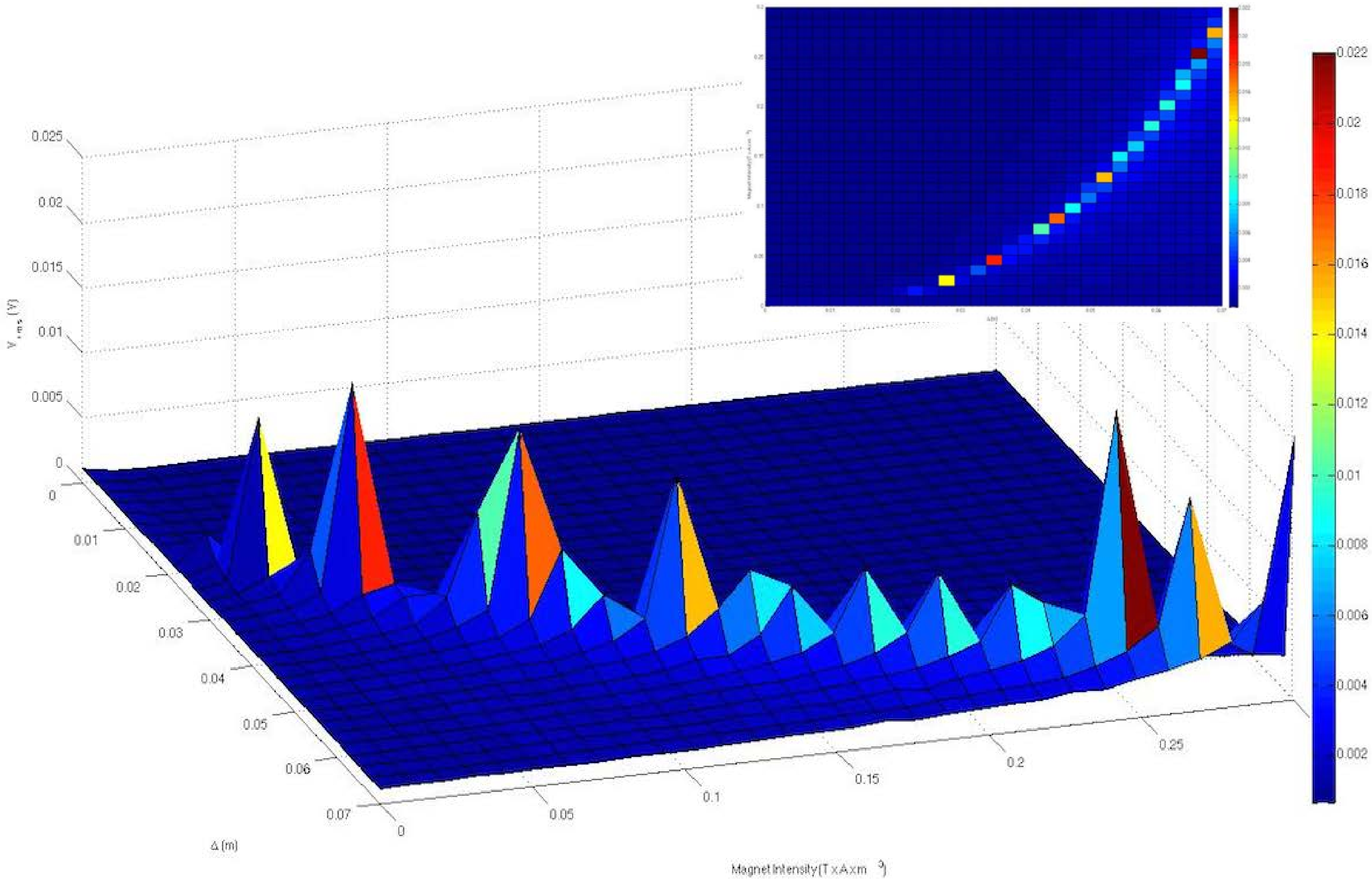
Methodology

- Plot the surfaces with both varying and constant force applied
- Derive a way to find, for a given Δ, the Q that maximizes the V_{rms}
- Find the pair (Δ, Q) that results in a global maximum V_{rms}

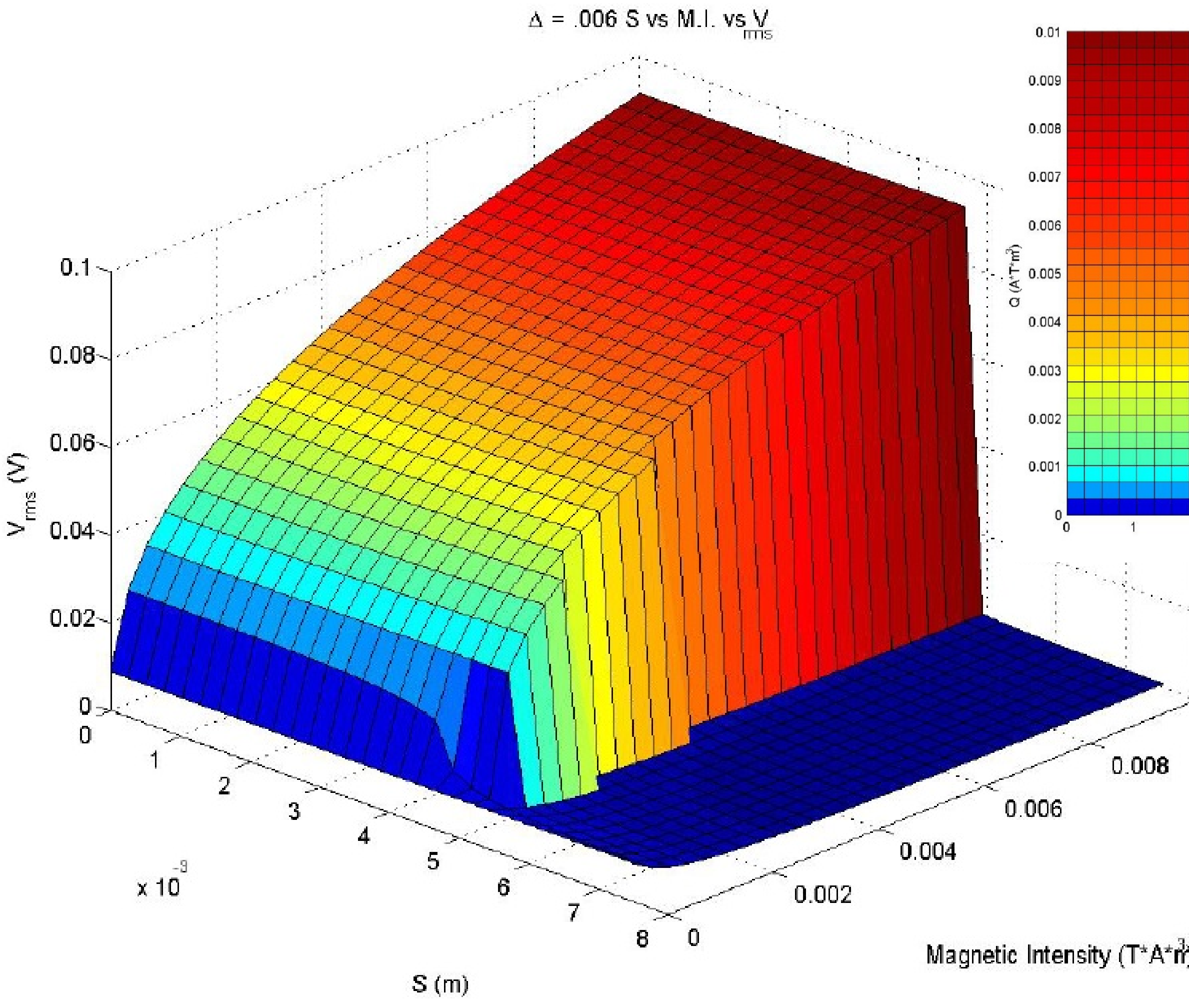
Single Magnet: Constant Force

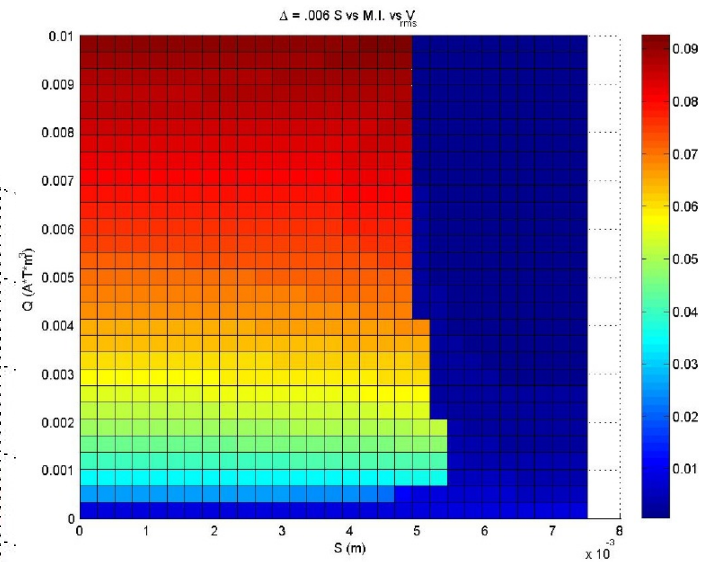


Single Magnet: Random force

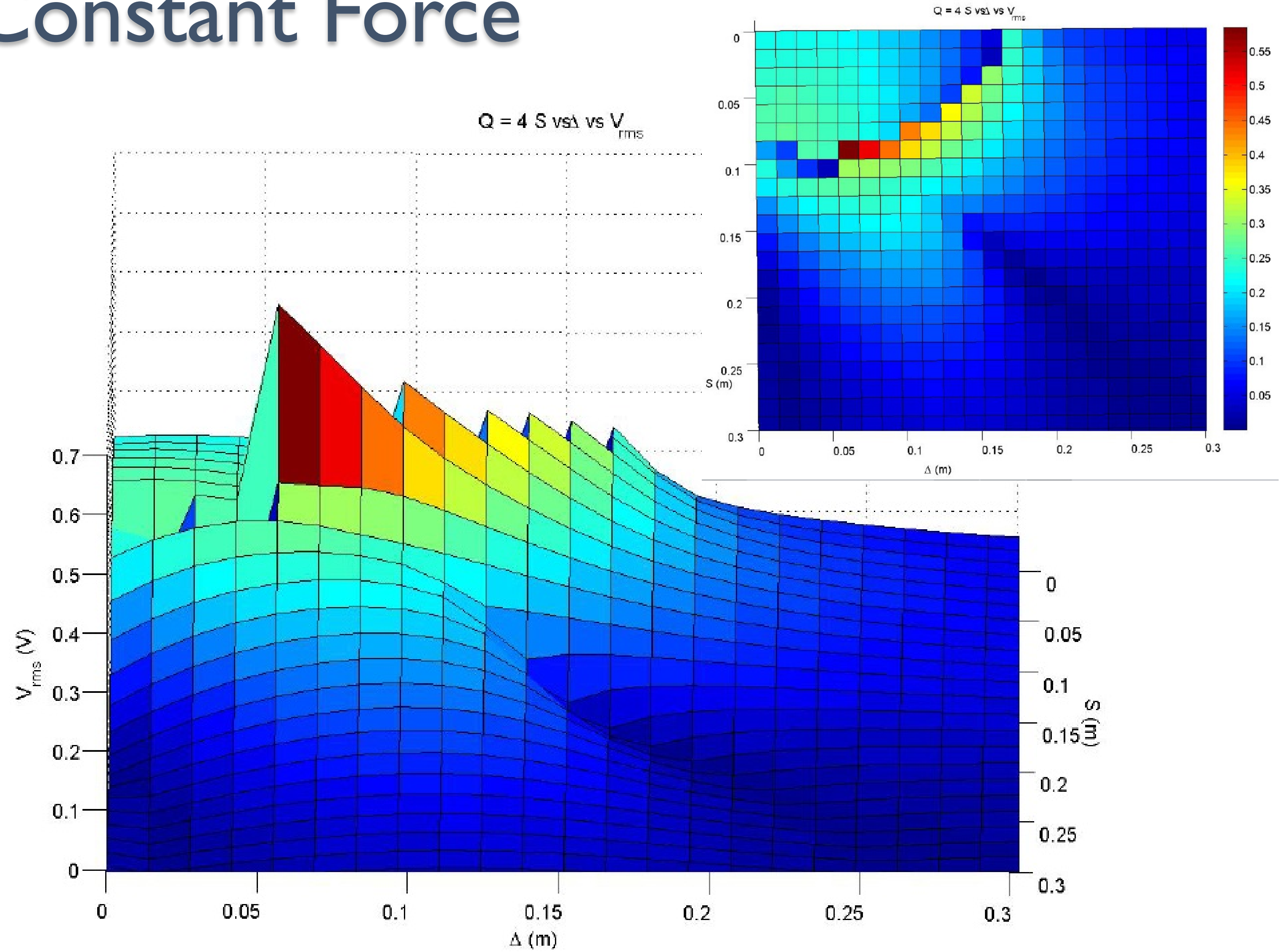


Double Magnet: Δ = Constant, Constant Force



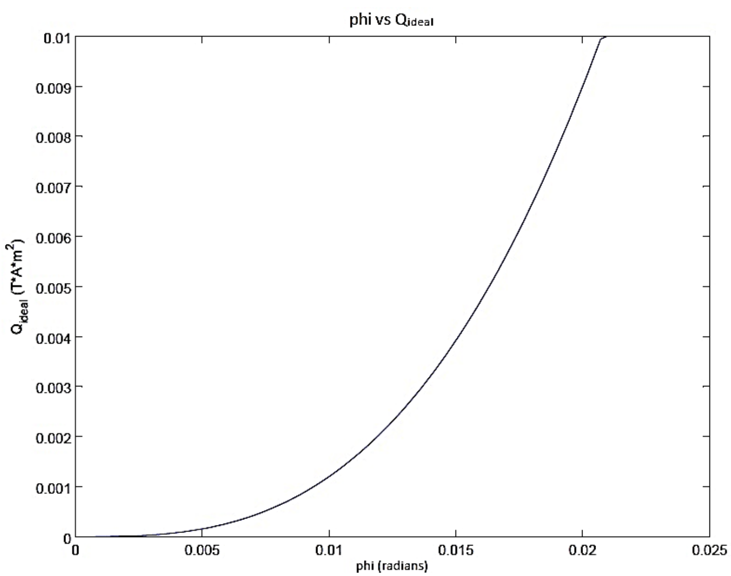


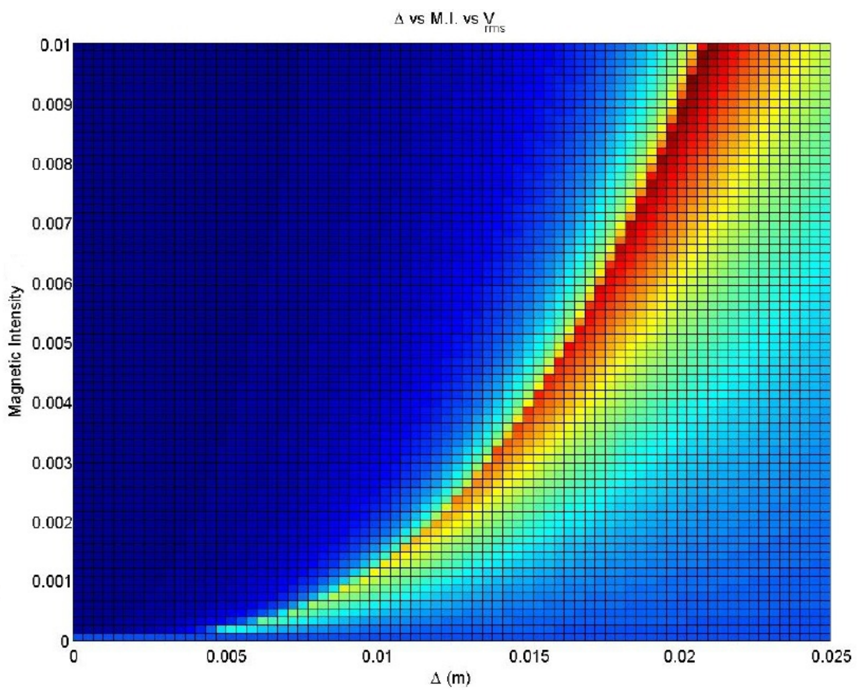
Double Magnet: Q = Constant, Constant Force



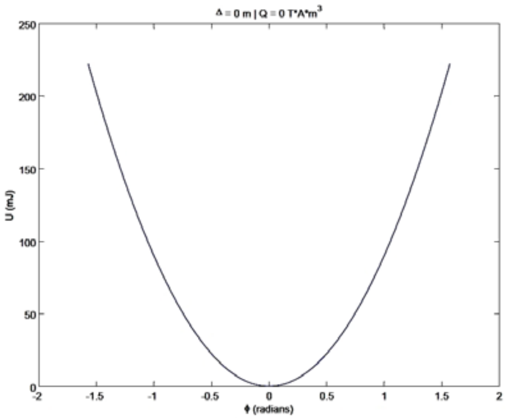
Δ vs Q_{ideal}

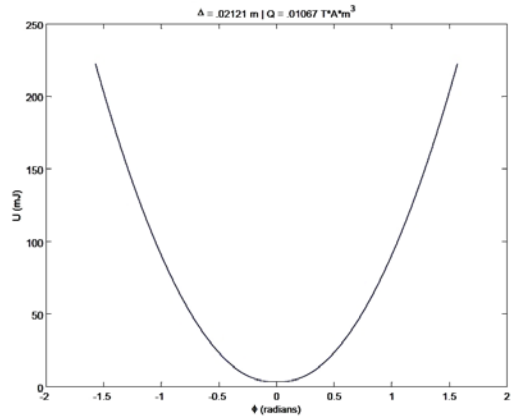
$$U(0) = U(\varphi_{min})$$

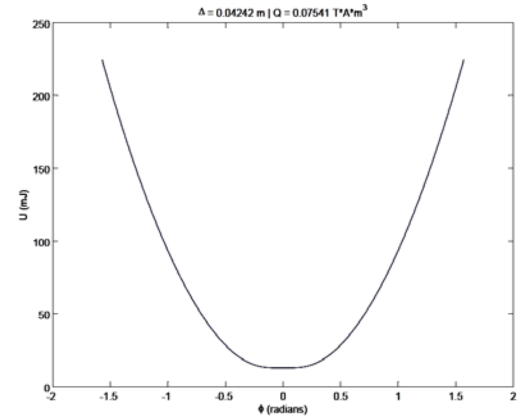


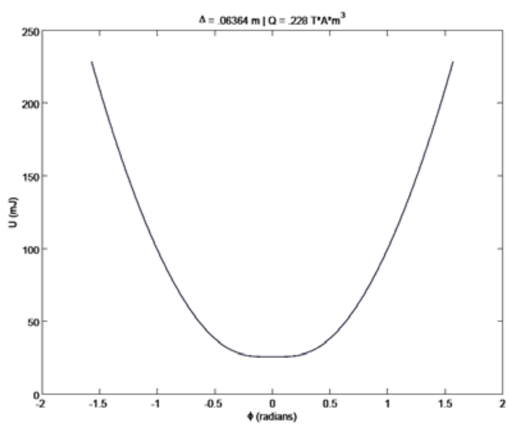


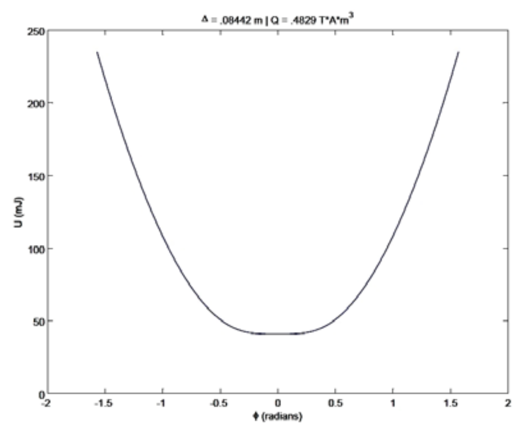
Results, Single Magnet : Potentials of Points Along Δ vs. Q_{ideal}











Result, Single Magnet: Along Δ vs. Q_{ideal} Curve

Δ (m)	**Q (T*A*m^3)**	**$V_{rms, ave}$ (V)**	**Standard Deviation**
0	0	.0012	.000011552
.007071	.000435	.0044	.00020404
.01414	.003315	.0062	.00029936
.02121	.01067	.0071	.00043506
.02828	.02423	.0082	.00048111
.03535	.04541	.0091	.00052889
.04242	.07541	.0099	.00049762
.04949	.1153	.0108	.00067839
.05657	.1659	.0108	.00064192
.06364	.228	.0114	.0007905
.07	.2942	.0117	.00064062
.07739	.384	.0121	.00078317
.08442	.4829	.0128	.00067265

Results, Single Magnet: Deviating from Δ vs. Q_{ideal}

Δ (m)	Q (T*A*m^3)	V_{rms} (V)	Standard Deviation
.07	.2942	.0117	.00064062
.07	.2992	.0232	.0051
.07	.2982	.0076	.0005606
.065	.2942	.0019	.00018046
.075	.2942	.0028	.00023107

Vrms(.07, .2942) < Vrms(.07, .2992)

Results, Double Magnets: Varying S

Δ = .07 m, Q = .2992 (T*A*m^3)

S (m)	V_{rms} (V)	Standard Deviation
0	.0232	.0051
.0005	.0211	.0041
.001	.0234	.0045
.005	.0109	.00063975
.01	.0049	.00062362
.05	.0013	.00011549

Conclusions

- The system is self-averaging
- A single magnet oscillator with a single-well potential produces a greater V_{rms} than single magnet, double-well potential oscillators and no magnet oscillator
- A relationship between Δ and Q_{ideal} is an approximation
- There is no ideal set of Δ and Q that will maximize V_{rms}

Future Work

- Determine the relationship between Δ and Q that maximizes V_{rms}
- Describe the V_{rms} of double magnet systems in terms of Q, Δ, and S.
- Alter the stochastic force to better approximate vibrations from walking, driving, wind, etc